

Probability and Random Processes

ECS 315

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

3 Classical Probability



Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday 14:00-15:30

Friday 14:00-15:30

Real coins are biased

- From a group of Stanford researchers

DYNAMICAL BIAS IN THE COIN TOSS

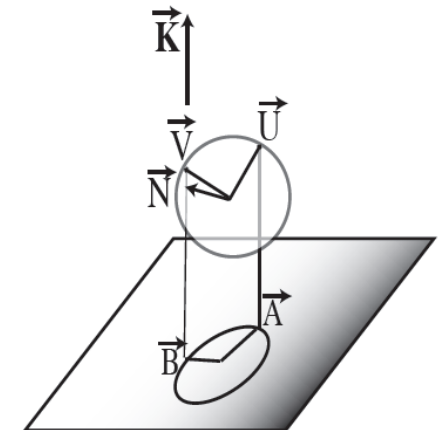
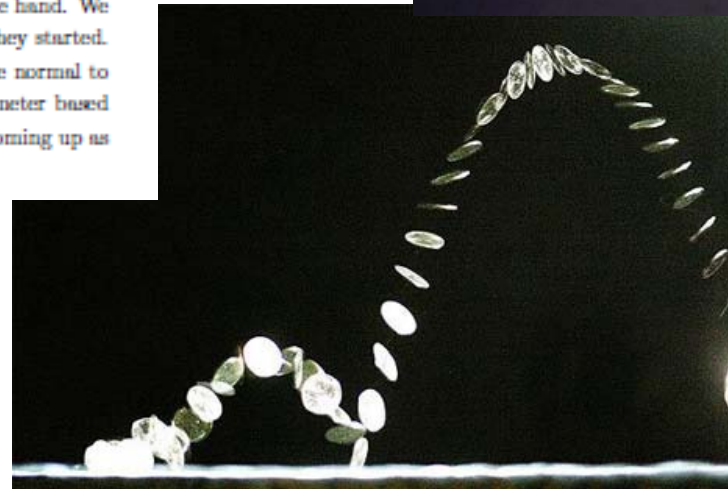
Persi Diaconis
Departments of Mathematics
and Statistics
Stanford University

Susan Holmes
Department of Statistics
Sequoia Hall
Stanford University

Richard Montgomery
Department of Mathematics
University of California
Santa Cruz

Abstract

We analyze the natural process of flipping a coin which is caught in the hand. We prove that vigorously-flipped coins are biased to come up the same way they started. The amount of bias depends on a single parameter, the angle between the normal to the coin and the angular momentum vector. Measurements of this parameter based on high-speed photography are reported. For natural flips, the chance of coming up as started is about .51.



The word “dice”

- Historically, **dice** is the plural of **die**.
- In modern standard English, **dice** is used as both the singular and the plural.



Example of 19th Century bone dice



The dice myth: “I always roll more ones”

Gaming Dice

- Cheaply made.
- Have rounded edges
- **Pips**
 - Indentations on the side of dice (the little dots cut out from each side) to indicate the value of the face
 - This produces an uneven balance as the “six” side has more pips (less material/weight) than the “one” side.
- **1s are by far the most common result**
 - An experiment using Chessex and GW dice got 29% ones on average.
- The amount of plastic saved from rounding the corners and hollowing out the pips of 2 dice actually gave them enough left over plastic to make a 3rd dice.





The three-dimensional dynamics of the die throw

M. Kapitaniak^{1,2}, J. Strzalko¹, J. Grabski¹ and T. Kapitaniak¹

[+ VIEW AFFILIATIONS](#)

Chaos **22**, 047504 (2012); <http://dx.doi.org/10.1063/1.4746038>

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Abstract

Full Text

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Data & Media

Metrics

Related

A three-dimensional model of a die throw which considers the die bounces with dissipation on the fixed and oscillating table has been formulated. It allows simulations of the trajectories for dice with different shapes. Numerical results have been compared with the experimental observation using high speed camera. It is shown that for the realistic values of the initial energy the probabilities of the die landing on the face which is the lowest one at the beginning is larger than the probabilities of landing on any other face. We argue that non-smoothness of the system plays a key role in the occurrence of dynamical uncertainties and gives the explanation why for practically small uncertainties in the initial conditions a mechanical randomizer approximates the random process.

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Key Topics

Friction

7.0

Chaos

6.0

Cameras

4.0

Chaotic dynamics

4.0

Classical mechanics

4.0



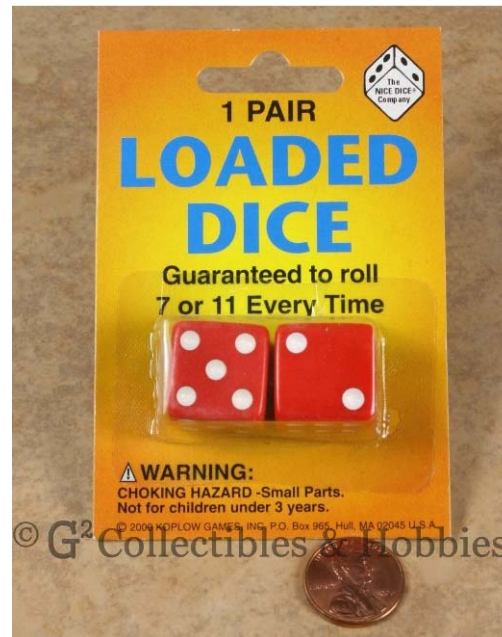
Non-Symmetry in Dice

ลูกเต๋าคพบในเมือง
โบราณอุทอง
จังหวัดสุพรรณบุรี
จัดแสดงใน
พิพิธภัณฑสถาน
แห่งชาติ อุทอง



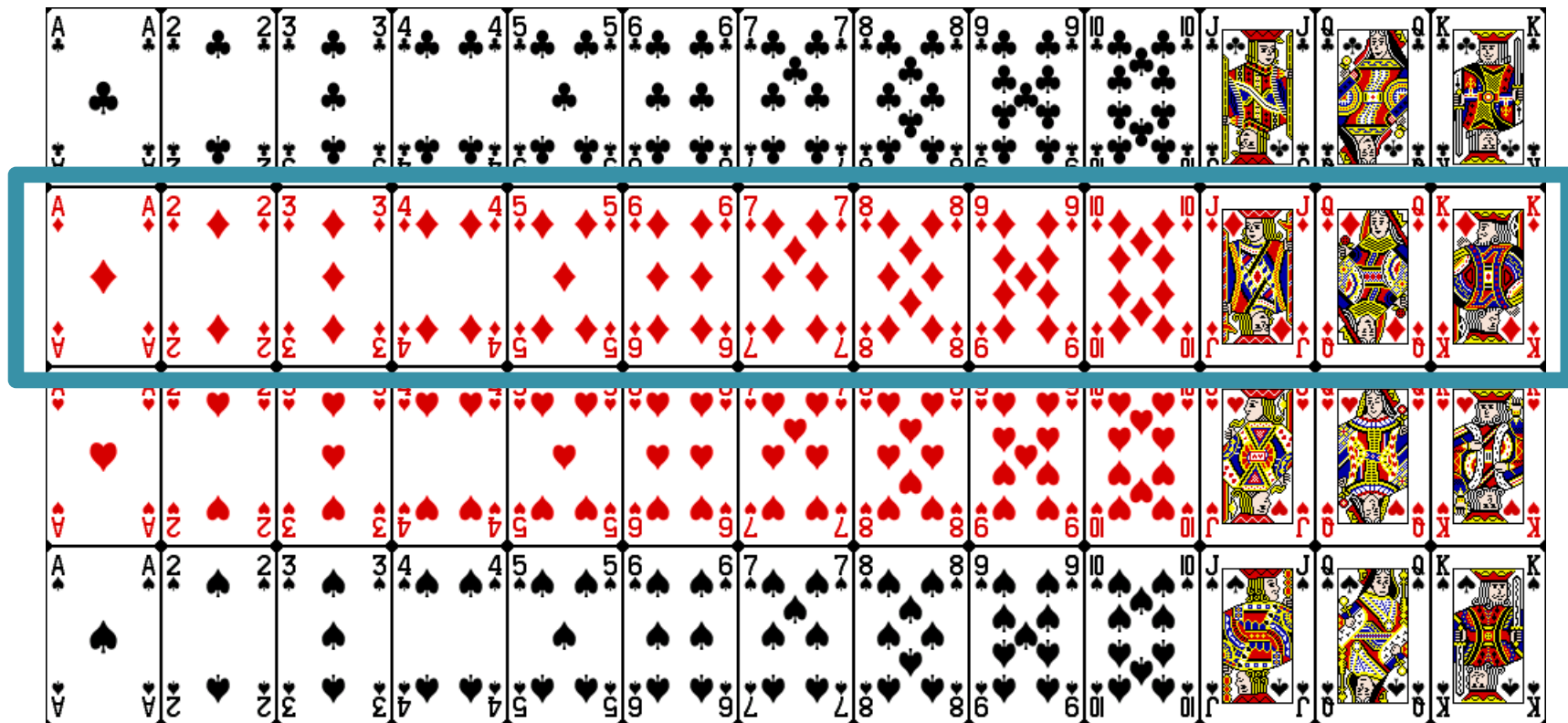
Loaded Dice

- A loaded, weighted or crooked die is one that has been tampered with so that it will land with a specific side facing upwards more or less often than a fair die would.



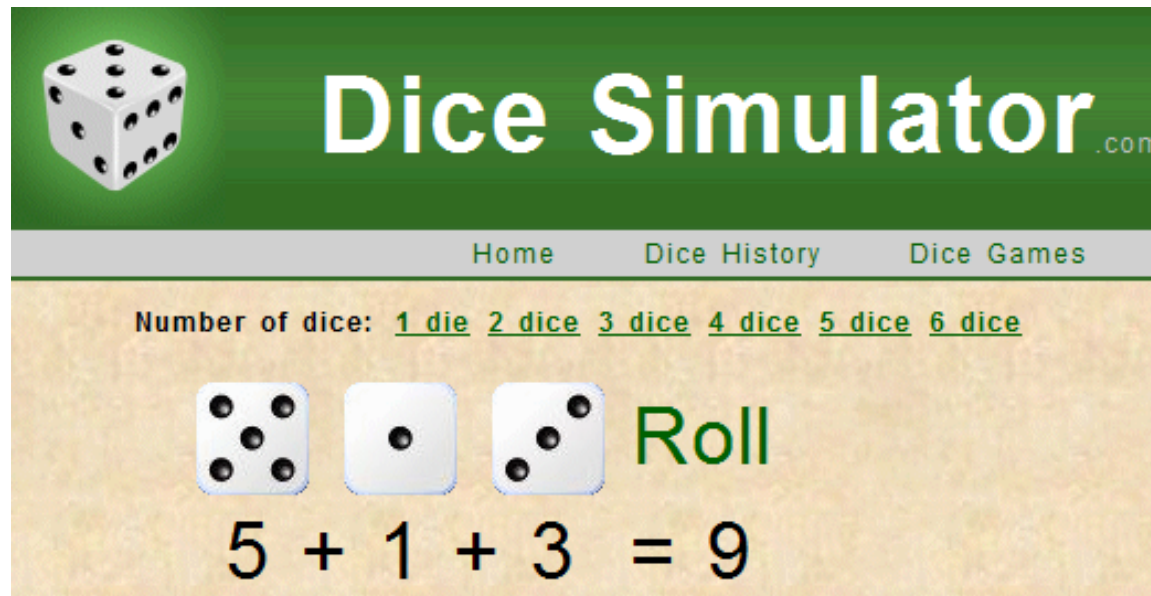
Example

- In drawing a card from a deck, there are 52 equally likely outcomes, 13 of which are **diamonds**. This leads to a probability of $13/52$ or $1/4$.

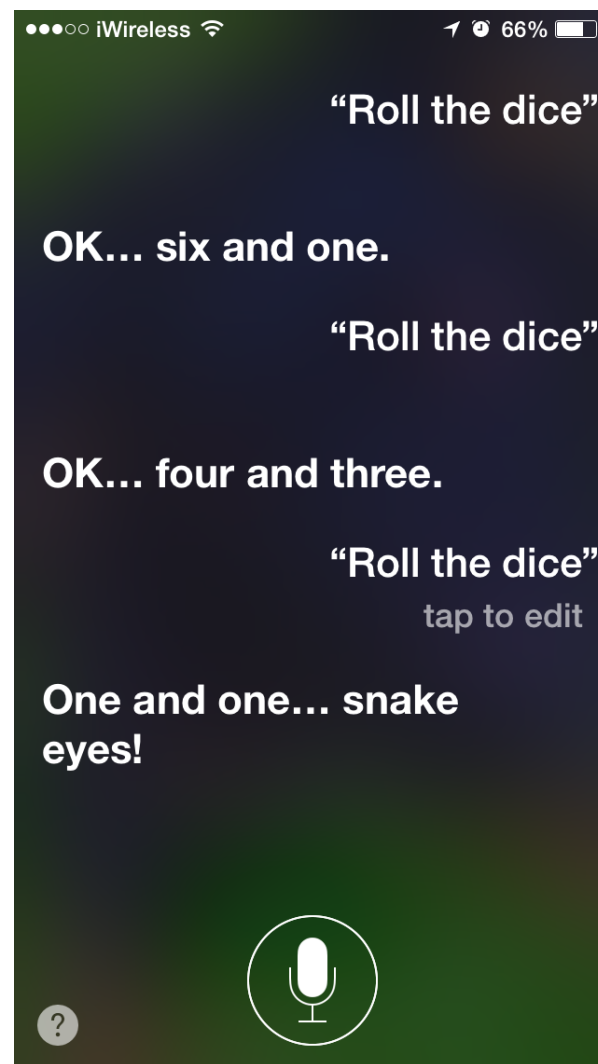
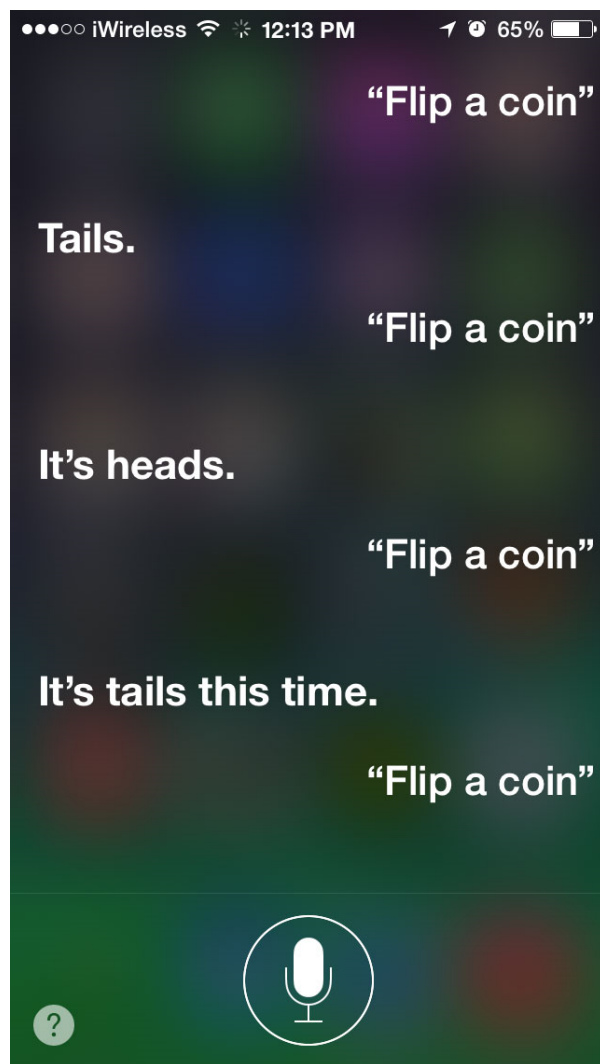


Dice Simulator

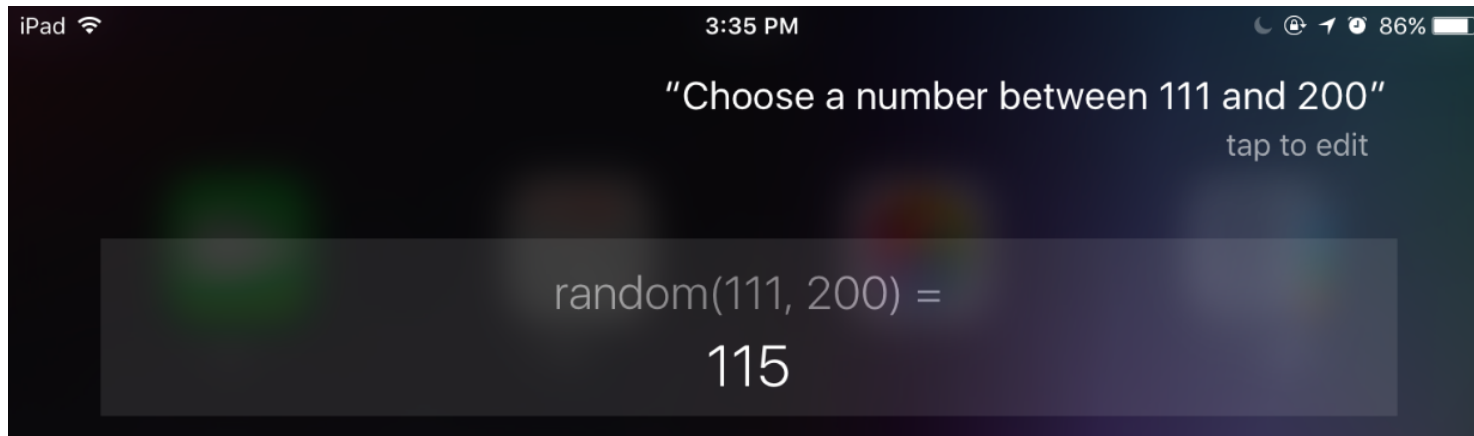
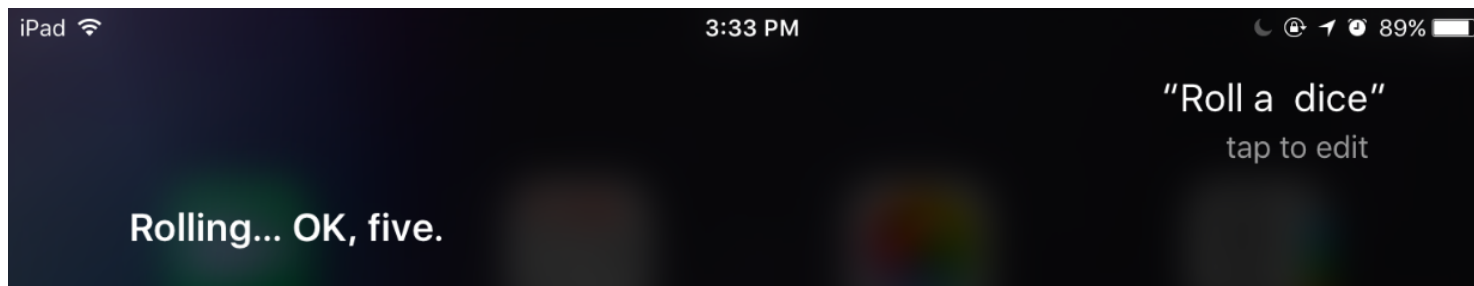
- <http://www.dicesimulator.com/>
- Support up to 6 dice and also has some background information on dice and random numbers.



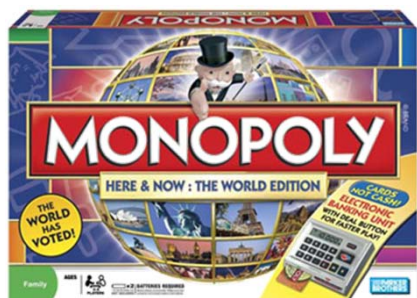
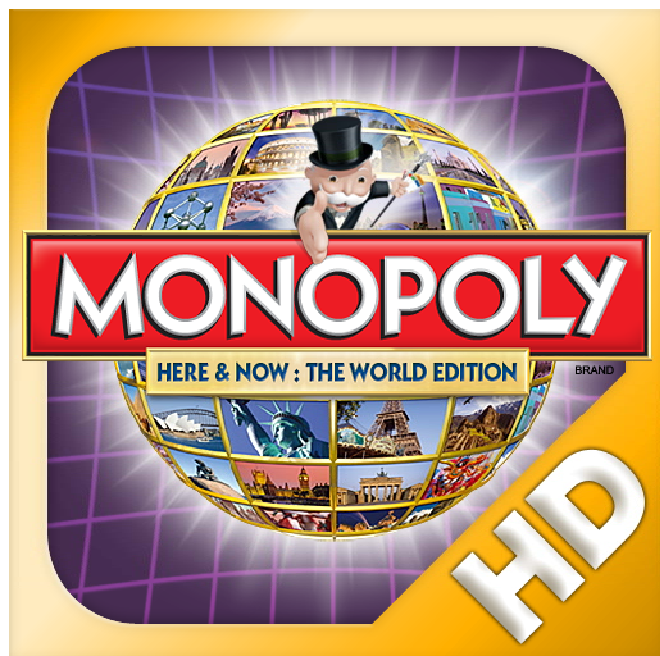
Roll the dice or flip a coin with Siri



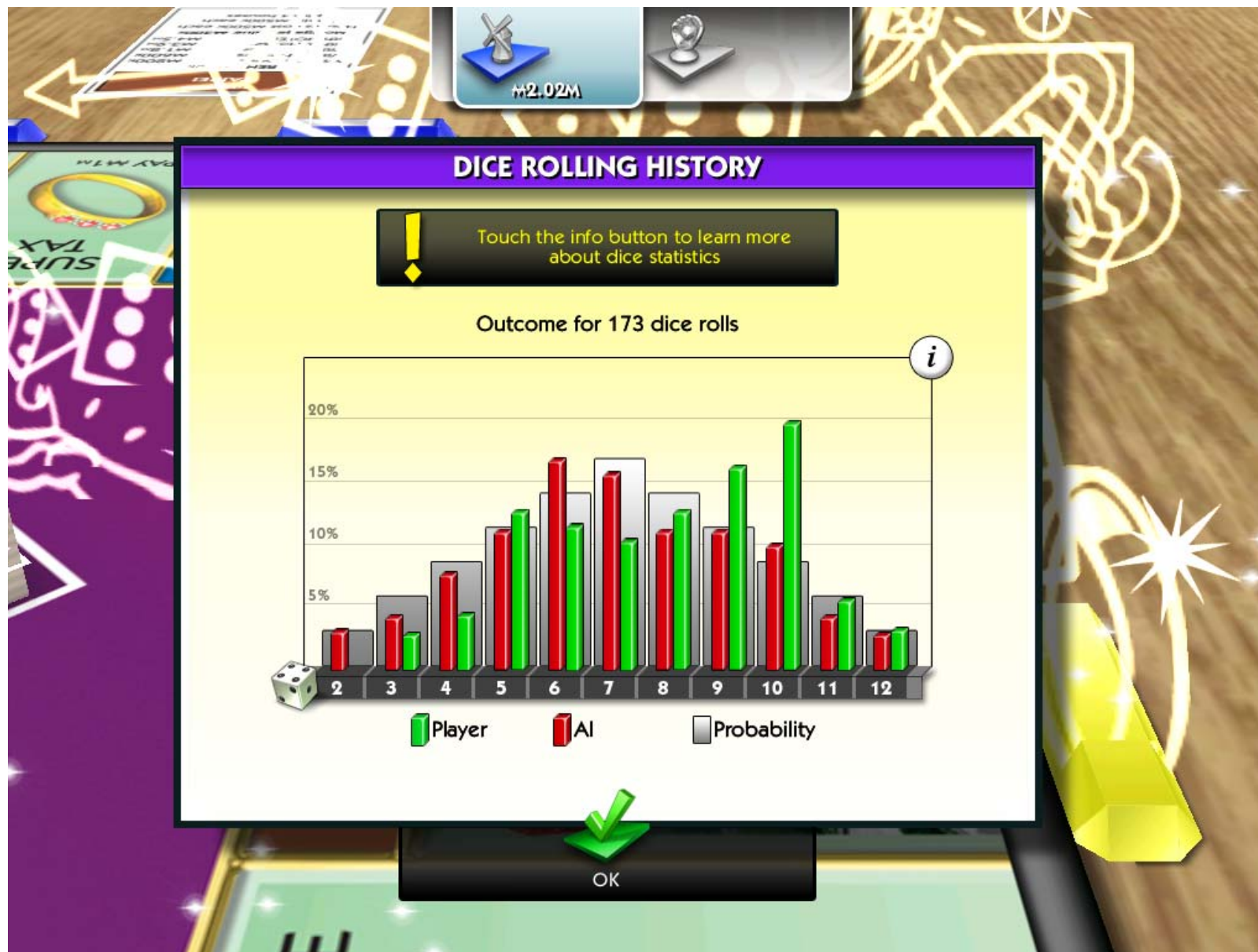
Do more with Siri



Two Dice

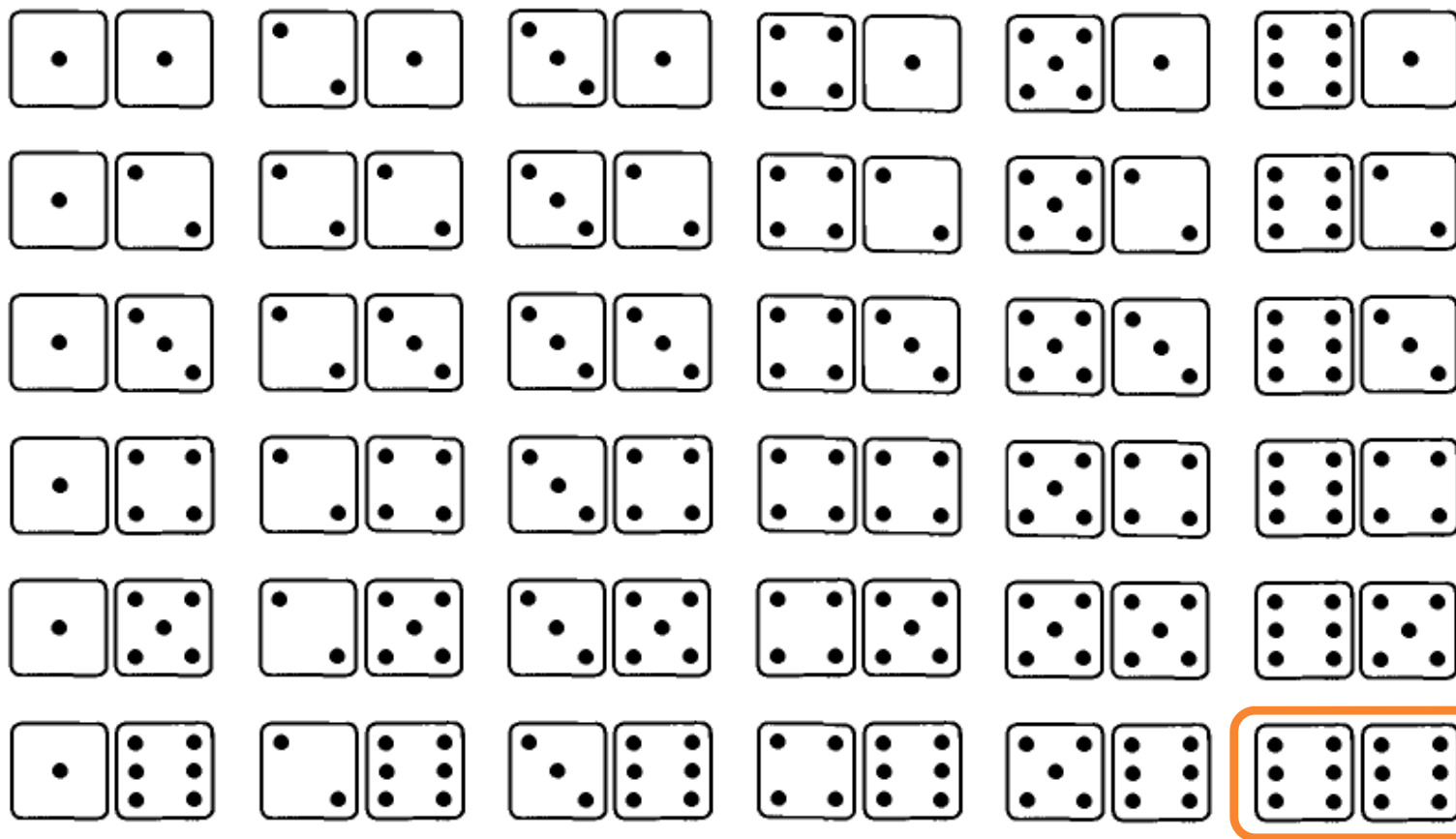


Two-Dice Statistics



Two Dice

- A pair of dice




Double six

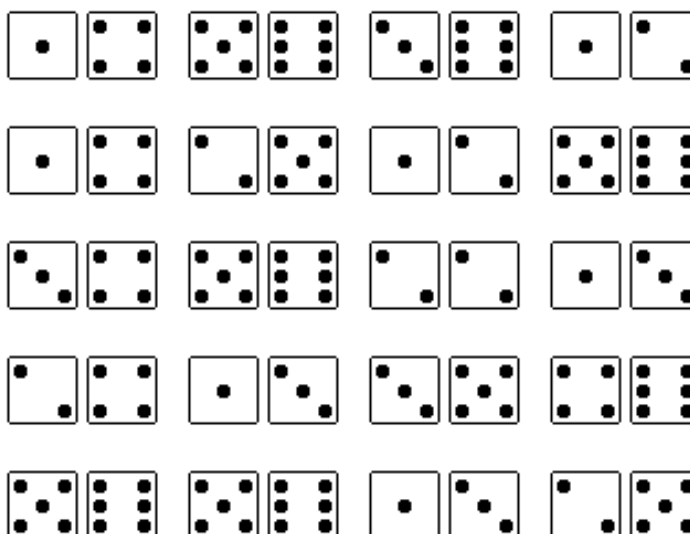


Two dice: Simulation



 <i>Simulated Experimental Dice-Roll Data (2 dice)</i>
Roll how many sets of 2 Dice? <input type="text" value="20"/> <input type="button" value="Roll Them!"/>
The results of the dice rolls will appear in a pop-up window. If you have pop-ups disabled, you might have to check to see if another window opened in the background.
<input type="button" value="Reset Form"/>
©Jeff LeMieux, 2002

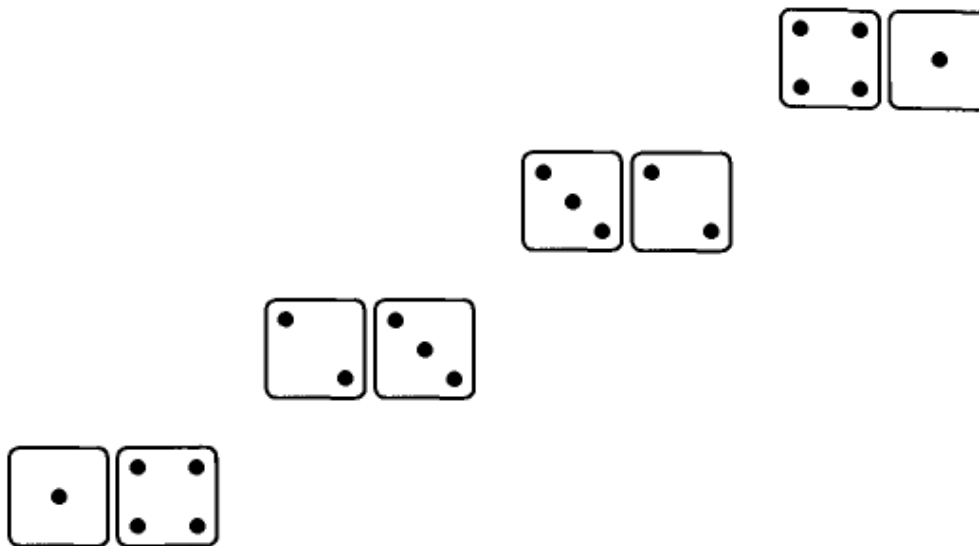
[<http://www2.whidbey.net/ohmsmath/webwork/javascript/dice2rol.htm>]



Two dice

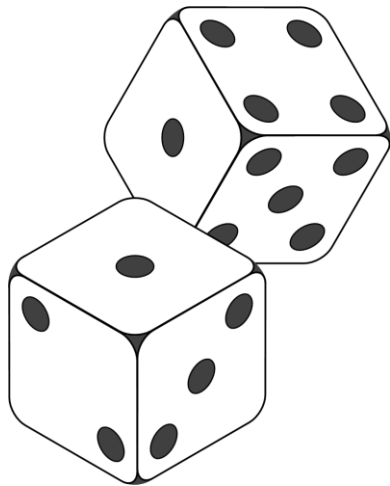


- Assume that the two dice are fair and independent.
- $P[\text{sum of the two dice} = 5] = 4/36$



Two dice

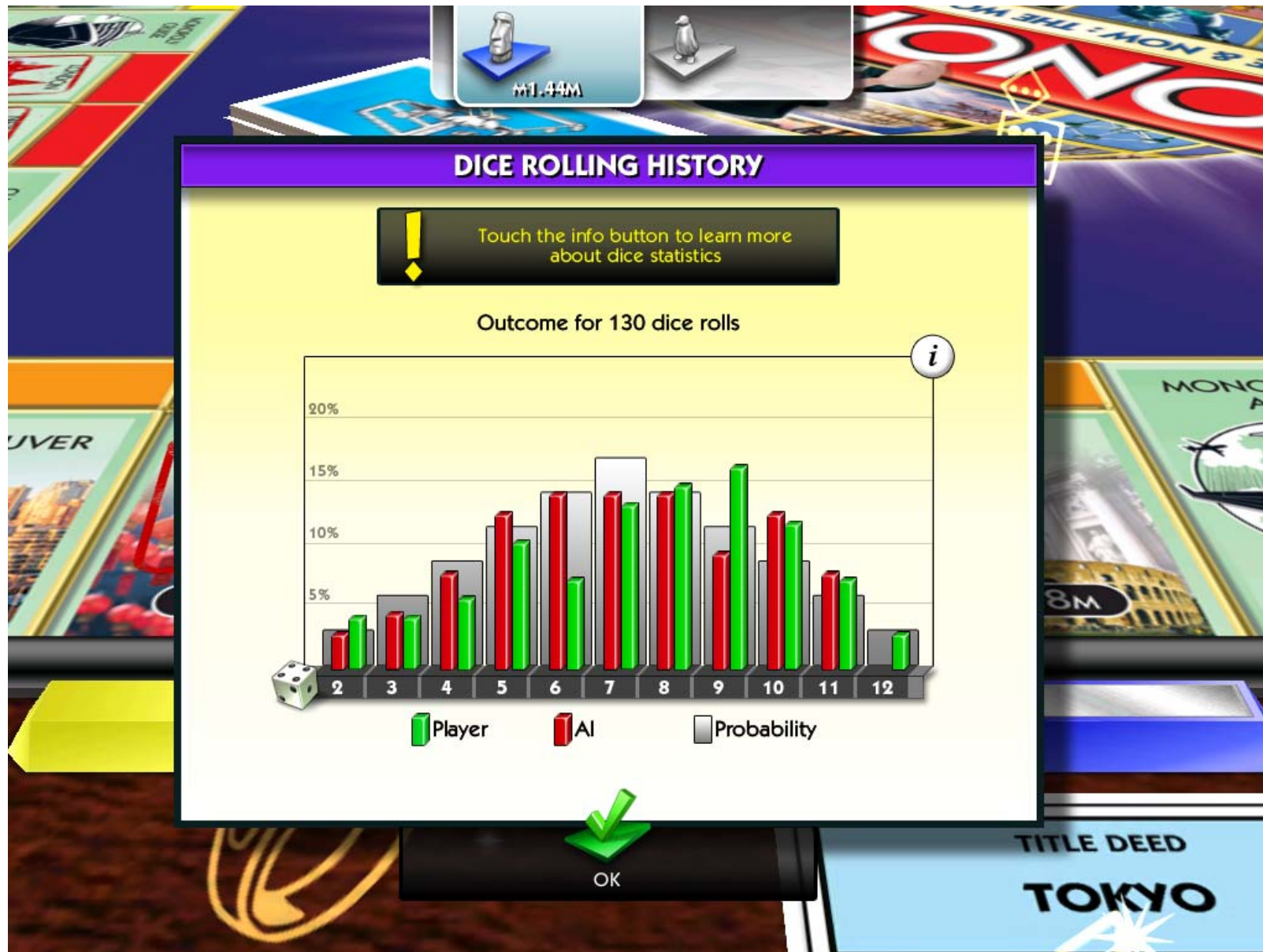
- Assume that the two dice are fair and independent.



DICE CHART		
ROLL		PROBABILITY ↗
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36

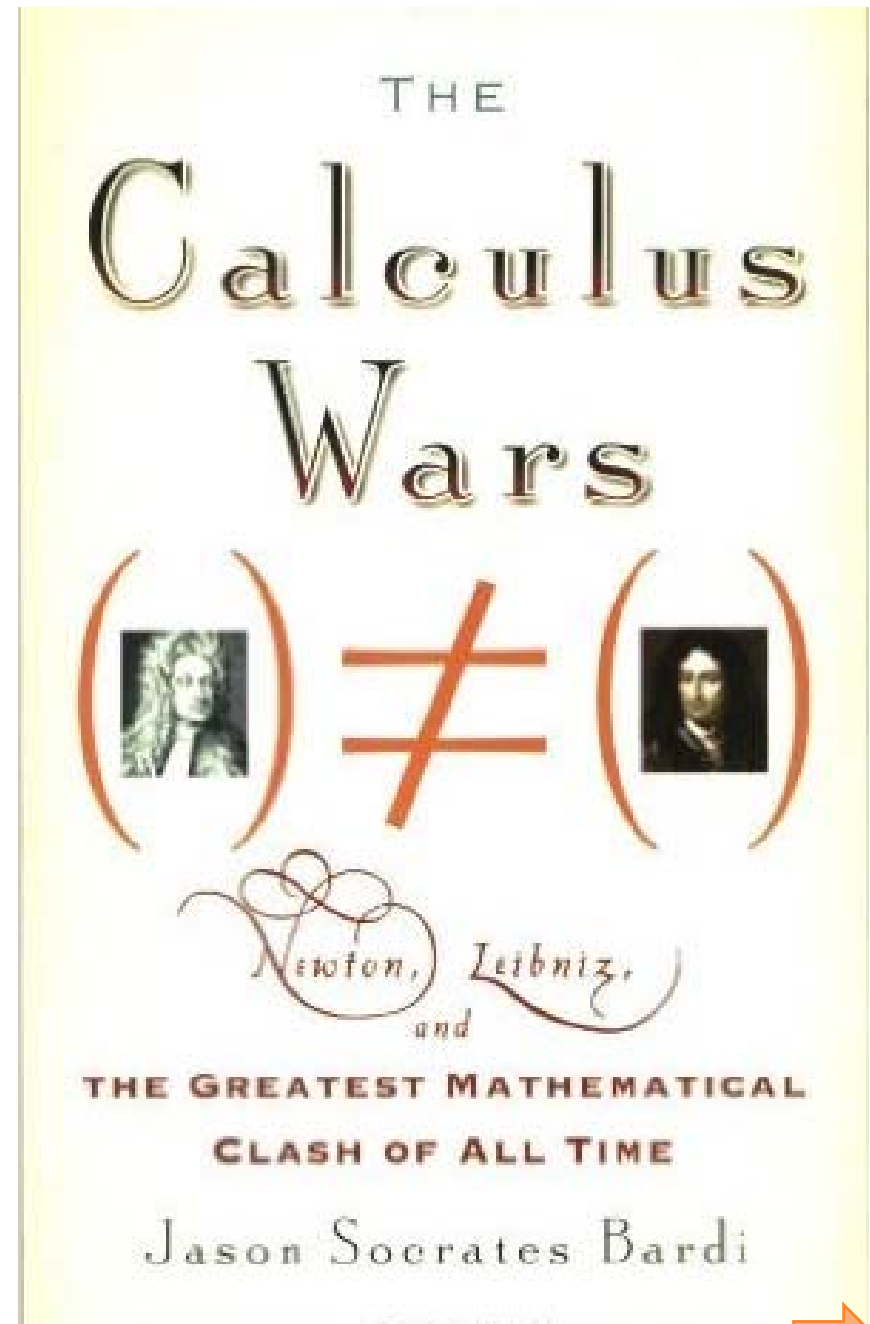
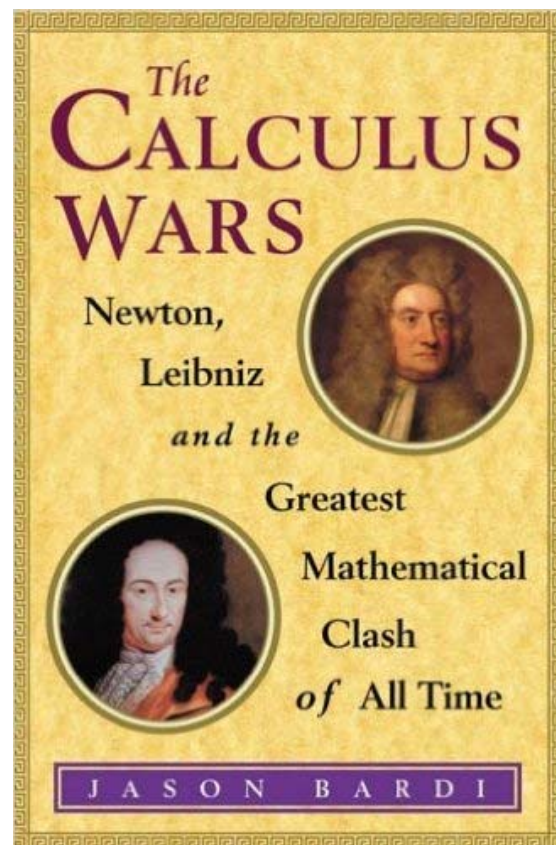


Two-Dice Statistics



Calculus War

- Nontechnical account of the battle between Newton and Leibniz over **who invented calculus**.



Calculus War



Calculus War: Leibniz

